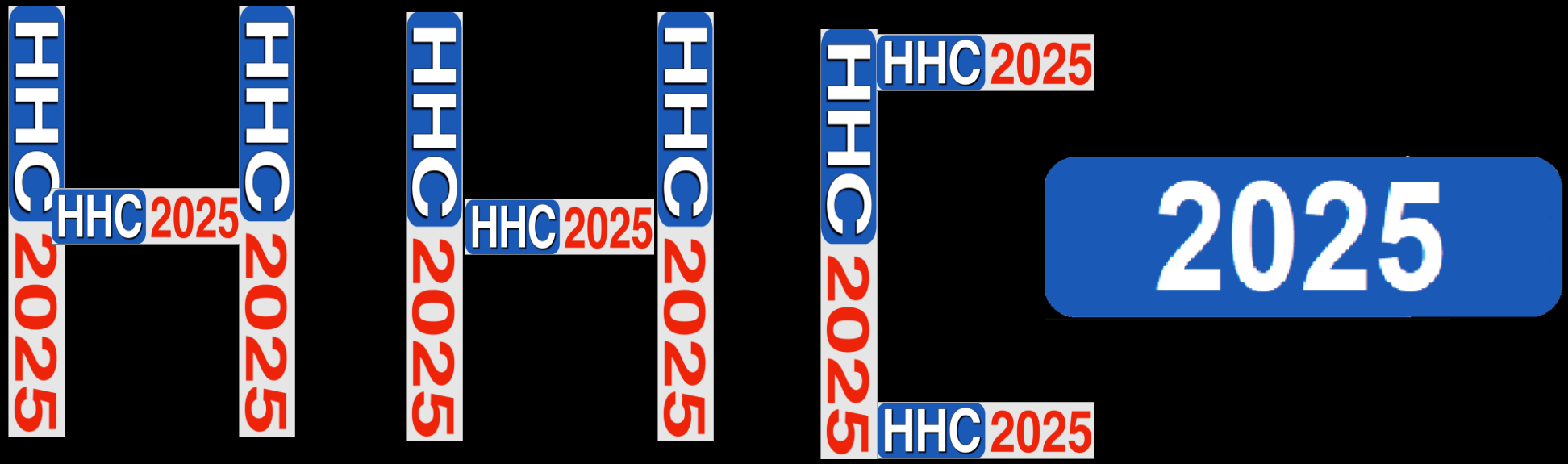
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The Next Prime

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V 2 Rev.2025 09 10

Sorry,

This is not about the

HP Prime computer

Number Theory: a branch of mathematics dealing with integers.

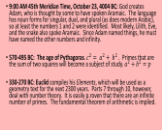
- Elementary Number Theory: uses algebraic properties of integers to prove theorems.
- Analytic Number Theory: uses properties of complex variable functions to prove theorems about prime integers.
- Computational Number Theory: uses computer to search for counterexamples of conjectures about primes and other integers. Our NEXTP function empoweres the curious to explore.

Historical Milestones

- **9:00 AM 45th Meridian Time, October 23, 4004 BC:** God creates Adam, who is thought by some to have spoken Aramaic. The language has noun forms for singular, dual, and plural (as does modern Arabic), so at least the numbers 1 and 2 were identified. Most likely, Lilith, Eve, and the snake also spoke Aramaic. Since Adam named things, he must have named the other numbers and infinity.

- **570-495 BC: The age of Pythagoras**. $c^2 = a^2 + b^2$. Primes that are the sum of two squares will become a subject of study. $a^2 + b^2 = p$

- **330-270 BC: Euclid** compiles his *Elements*, which will be used as a geometry text for the next 2300 years. Parts 7 through 10, however, deal with number theory. It is easily p.roven that there are an infinite number of primes. The fundamental theorem of arithmetic is implied.



Historical Milestones (continued)

- **276-194 BC: Eratothsenes** becomes head librarian at Alexandria Egypt (under Greek rule at that time). Accurately measures the circumference of the earth. Formulates the sieve method of finding prime numbers (now an exercise for beginning programming students).
- **The Dark Ages:** German armies ravage the Arabian peninsula. Not so dark if you lived in Baghdad prior to the Mongoloid pillage of 1258. Greek mathematical knowledge is preserved and finds its way to Fibonacci who converts Europe away from Roman numerals. Big events in 1400's throw world into turmoil.
- **1601-1665: Pierre de Fermat** formulated Fermat's little theorem, and the monster $i^n + j^n = k^n \Rightarrow n \leq 2$. This was finally proven after 350 years. Fermat's little theorem, however, is easy to prove.

Historical Milestones

- **1601-1665: Pierre de Fermat** formulated the monster $i^n + j^n = k^n \Rightarrow n \leq 2$. This was finally proven after 350 years. Fermat's little theorem, is easy to prove: $\boldsymbol{a^p (mod\ p) = a}$

- **1623-1662 Pascal** builds the first calculator and invents the triangle. There is something about the prime rows of the Pascal triangle is not true for other rows.

More Historical Milestones

- **1690-1764 Golbach:** Conjectures in 1740 that every even number greater than 2 is the sum of two primes. This has never been proven.

- **1707-1783 Leonardt Euler** invents the totient function, i, π, and $n^2 - n + 41$, which generates 40 primes. Finds that every prime p of the form 4n+1 is the sum of two squares.

- **1752-1833 Adrien-Marie Legendre** conjectures that there is at least one prime between n^2 and $(n+1)^2$. It has been tested into the bozillions by computer, like the Goldbach conjecture, but never proven. Prime number theorem conjectured in 1798, proved in 1896.

Historical Milestones (almost done)

- **1736-1813 Joseph Louis Lagrange:** Kept his head during interesting times.
- **1777-1855 Karl Gauss:** Conjectures the prime number theorem in 1793, proven 100 years later.

Historical Milestones

- **1826-1866 Rieman:** Creates the Rieman zeta function, which can be expressed two ways. This connects the study of prime numbers to the analysis of functions. $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = [\prod_{n=1}^{\infty}(1 - {p_n}^{-s})]^{-1}$. The Rieman hypothesis is still unproven. Many conjectures about primes depend on it.
- **1821-1894 Chebyshev** discovered the Chebyshev bias in prime numbers modulo 4.

More on the Distribution of Primes

- 1822-1900 Joseph Bertrand conjectured in 1845 that there is at least one prime between *n* and *2n*. Proved by Chebyshev in 1852. Precisely stated as

$$\pi(n) - \pi\left(\frac{n}{2}\right) \geq 1 \text{ for all } n>1.$$

- Not Proven: There is a prime between $n^2 \; and \; (n+1)^2$ but is tested into the godzillions

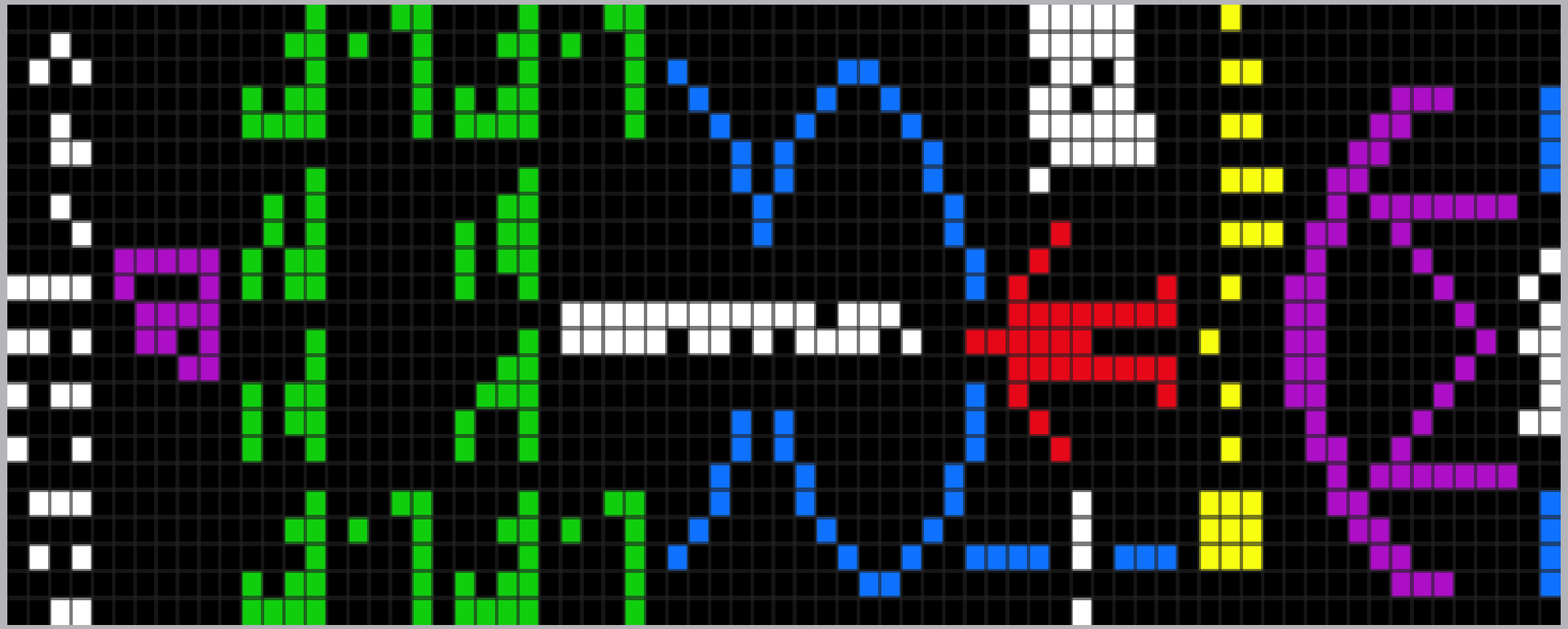
Modern Importance of Primes

Cryptographic systems are based on large primes.

Communication systems based on small primes encourage the recipient of a message to factor it into rows and columns. A good example is the Arecibo message, which may provoke an alien invasion when M-13 receives it in about 25000 years. *Science has not ruled out the possibility that they will come to do their usual to us.*

Prime Target is a thriller television series created by Steve Thompson and starring Leo Woodall as a mathematician. It premiered on Apple TV+ on 22 January 2025, consisting of 8 episodes. It is based on the importance of prime numbers in data security.

The Arecibo Message. See any mistakes?



NEXP Capabilities and Limitations

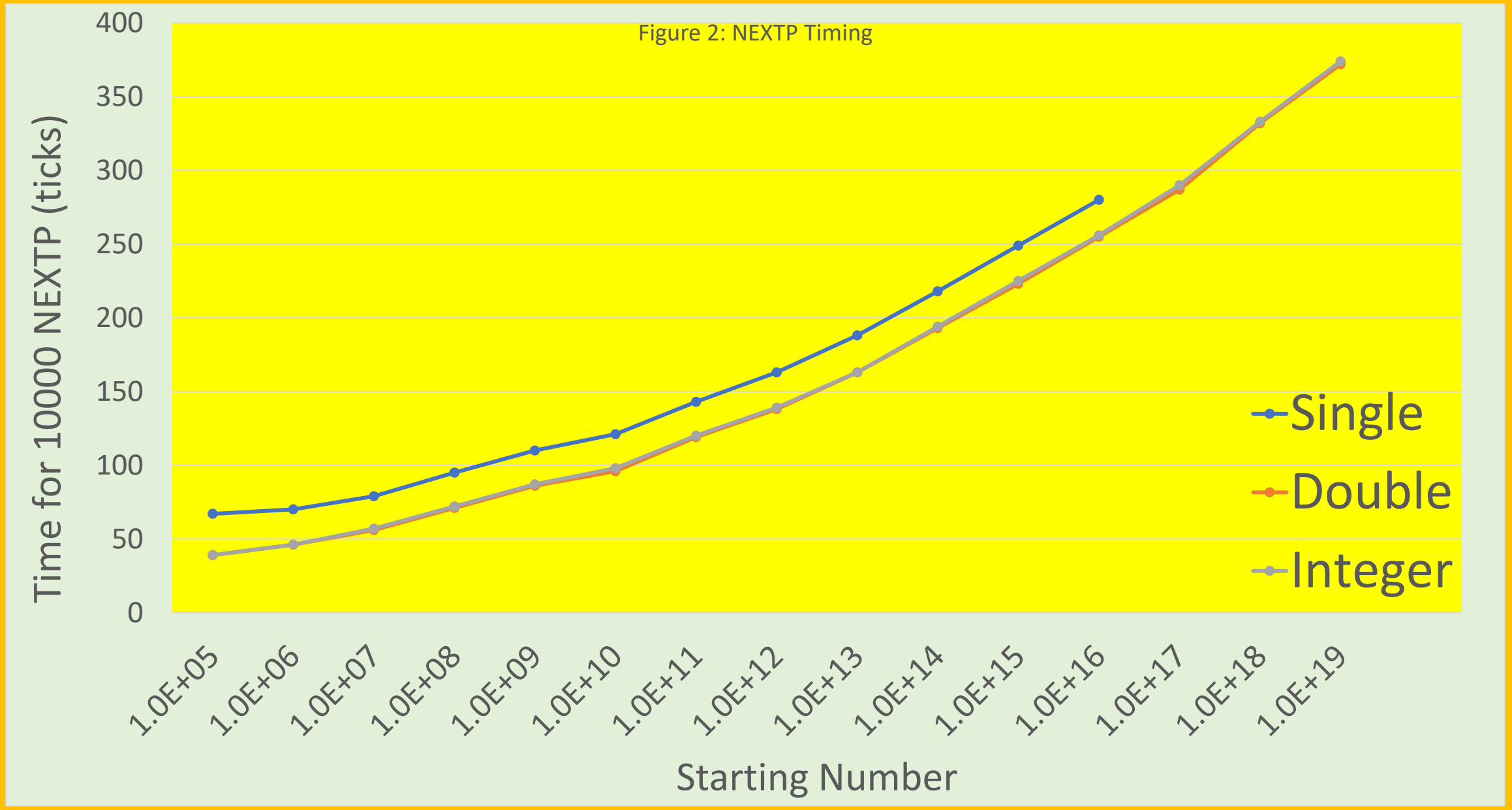
- Input Range
  - Single Precision, maximum prime is 9.999 999 999 999 917 E+15
  - Double Precision and Integer 7FFF FFFF FFFF FFE7, about 9.22337 E+18.
  - HP Prime: takes a while to compute $NextPrime(10^{2000}) = 10^{2000} + 4561.$ (I did not check this by hand.)

Something beyond the sieve of Eratosthenes is going on!

The sieve would require checking on the order of 4340000000000000000000000000000000000 0000000000000000000000000000000000000 00000000000000000000000000 prime divisors.

The sun will go out before that can happen.

NEXTP Timing



Some Functions Related to Prime Numbers

- $\pi(n)$ Number of primes less than or equal to n
- G(n) Prime gap
- g(e) Goldbach function (even numbers only)
- Φ(n) Euler totient function
- Z(z) Riemann zeta function

n	π(n) Count	Time (seconds)	$\frac{n}{Ln(n)-1}$	$\int_2^n \frac{dx}{Ln(x)}$
10	4	0	7.7	5.1
3.16E+02	65	0	66.4	70.0
1.00E+03	168	0.10	169.3	176.5
3.16E+03	446	0.31	447.7	461.4
1.00E+04	1229	0.62	1218.0	1243.9
1.00E+05	9592	6	9512.1	9622.5
1.00E+06	78498	53	78030.4	78596.4
1.00E+07	664579	531	661459.0	664735.9
1.00E+08	5761455	9549	5740303.8	5760909.2
3.16E+08	17071133	35988	17015545.7	17068398.8
1.00E+09	would tak≈30 hours.		50701542.4	50839001.8
1.00E+100	the sun will go out!		4.36189E+97	4.36197E+97

Prime Gap

This code developed by top-down structured object-oriented methodology in accordance with DOD-STD-2167.

LBL 'GAP'		
LBL A	(number-->gap)	
NEXTP		
RCL-L		
RTN	(gap)	

Different kinds of gaps

- Twin Primes (gap is 2)
- Prime Cousins (gap is 4)
- Sexy Primes (gap is 6)

UNSOLVED PROBLEM has been with us a long time: **are there an infinite number of twin primes?** (and its generalization)

Goldbach Function

- Number of prime pairs that add up to an even number E.
  - Example: g(2) = 0 (because 0 and 1 are not considered prime numbers)
  - Example: 14 = 3+11 = 7+7, so g(14)=2
  - Example 22 = 3+19 = 5+17 = 11+11, so g(22)=3
  - When testing primes, it is surprising how soon you find a pair.
- Great unsolved problem: $E > 2 \rightarrow g(E) \geq 1$.
  - Has been tested to very high numbers; no counterexample found
  - Leads to Richard's no return conjecture: $E > 12 \rightarrow g(E) \geq 2$.
  - Then $E > 68 \rightarrow g(E) \geq 3$. etc.
  - Conjecture #2: there is something about the no return values.

WHY CAN'T THEY DO IT?

- The Goldbach function grows into the thousands and beyond for large even numbers.
- It seems to reach certain minimum values that it stays above for greater even number arguments.
- These floors also grow to very large numbers.
- So why can't they prove that it stays above **zero** for $E > 2$? It should be easy!
- **This problem has been around since 1741.**

The Infemum Set of the Goldbach Function

- An even number N is an infimum point if g(N+k) > g(N) for all 0 < k < ∞.
- Some members of the infimum set are

N	g(n)	N	g(n)	N	g(n)	N	g(n)
12	1	1112	16	100094	570	1002002	3963
68	2	**1412**	**18**	100328	573	1002062	3972
128	3	1448	20	101342	574	1004842	3981
488	9	2936	31	103652	588	1013858	4016
632	10	2978	34	104102	592	1017026	4017
692	11	3092	35	106138	595	1020898	4025

Euler Totient Function ϕ(n)

- The number of integers k less than n for which GCD(n,k) = 1
- Important in number theory
- Must be programmed on WP-34, but built in to HP Prime.
- HP Prime algorithm much quicker than counting. Euler(10^1000) seems instantaneous! How do they do it?

Riemann Zeta Functin ζ(z)

(formulated by Euler around 1730)

- $\zeta(s) = \sum_{k=1}^{\infty} \frac{1}{k^s}$
- Note that for s=1 it is the harmonic series and diverges
- Note that for s = 2 it is the reciprocal square series, also known as the Basel problem. The reciprocal of this number is the probability that two random integers are relatively prime
- The connection to prime number theory is due to

$\sum_{k=1}^{\infty} \frac{1}{k^s} = \prod_{p\ prime} \frac{1}{1-p^{-s}}$, which was proved by Euler.

E

Darn it!

The Rieman zeta in the WP-34s is limited to real numbers. You can't investigate the interesting strip.

Another Unsolved Problem

- **Riemann hypothesis** of 1859 is the conjecture that the Riemann zeta function has its zeros only at the negative even integers and complex numbers with real part 1/2.
- They're still working on it.

WARNING!

- "Many of these problems have eluded solution for centuries. Students should be forewarned that attempts to setle such problems are often time consuming and futile. "
- But, when did any of us stop being curious?

Rosen, Kenneth H., Elementary Number Theory and its Applicatins, 3rd ed., Addison-Wesley, 1993. page viii.

Imagine...

You and your calculator pass through a hyperspace wormhole into the seventeenth century and you encounter Fermat, Pascal, and Goldbach.

- How could you evade the Holy Inquisition, change history and advance number theory a couple of centuries?
- What computational miracles could you perform with your remaining battery charge that would be of value to those pioneers and save them years of laborious hand calculations?

- **Amazingly little is known about prime numbers and how they are distributed.**
- **There is work to do. Hundred year old problems remain unsolved!**
- **We have better tools at our disposal.**

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OK, you can all wake up now (if possible). And be curious!

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